IP-1552

M. A. / M. Sc. (First Semester) Examination, Dec. 2021

MATHEMATICS

Paper : Second

(Real Analysis)

Time Allowed : Three hours

Maximum Marks : 50

Note : *Attempt questions of all* ***two*** *sections as directed.*

Section-'A'

(Short Answer Type Questions) **5×4=20**

Note : *Attempt all the five questions.* ***One*** *question from each unit is compulsory. Each question carries 04 marks.*

Unit-I

1. Define Riemann-stieltjes integral of a function f with respect to a monotorically increasing function α over $[a,b]$.

Or

If P^* is a refinement of P, then prove that

$$L(P, f, \alpha) \leq L(P^*, f, \alpha)$$

Unit-II

2. Define Rectifiable curve.

Or

If $\sum a_n$ is a series of complex numbers which converges absolutely, then prove that every rearrangement of $\sum a_n$ converges and they all converge to the same sum.

Unit-III

3. Using M-test show that the following series is uniformly convergent on $\mathbb{R}$

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + x^2}$$

Or

Apply Dirichlet's test to show that the following series is convergent on $\mathbb{R}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + x^2}$$

Unit-IV

4. Explain chain rule for differentiation of composite functions.

Or

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{for } (x, y) \neq (0, 0) \\ 0, & \text{for } (x, y) = (0, 0) \end{cases}$$

Show that $(D_{21}f)(0,0)$ and $(D_{12}f)(0,0)$ are not equal.

PTO

Unit-V

5. Explain Jacobians.

Or

Give a precise statement of Stoke' theorem.

Section-'B'

(Long Answer Type Questions) **5×6=30**

Note : *Attempt all the five questions.* ***One*** *question from each unit is compulsory. Each question carries 06 marks.*

Unit-I

6. State and prove fundamental theorem of calculus.

Or

If $f_1 \in \mathbb{R}(\alpha)$ and $f_2 \in \mathbb{R}(\alpha)$ on $[a, b]$ then show that $f_1 + f_2 \in \mathbb{R}(\alpha)$.

Unit-II

7. For $0 < x < 2$, $a_n(x) = \dfrac{2nx}{nx + 3}$. Show that $\{a_n(x)\}_{n=1}^{\infty}$

Unit-V

5. Explain Jacobians.

Or

Give a precise statement of Stoke' theorem.

Section-'B'

(Long Answer Type Questions) 5×6=30

Note : *Attempt all the **five** questions. **One** question from each unit is compulsory. Each question carries 06 marks.*

Unit-I

6. State and prove fundamental theorem of calculus.

Or

If $f_1 \in \mathbb{R}(\alpha)$ and $f_2 \in \mathbb{R}(\alpha)$ on $[a, b]$ then show that

$f_1 + f_2 \in \mathbb{R}(\alpha)$.

Unit-II

7. For $0 < x < 2, a_n(x) = \dfrac{2nx}{nx+3}$. Show that $\{a_n(x)\}_{n=1}^{\infty}$

is pointwise convergent but not uniformly convergent on $E = (0, 2)$.

Or

State and prove Riemann theorem.

Unit-III

8. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathbb{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, \ldots\ldots$, and suppose $f_n \to f$ on $[a, b]$, then prove that $f_n \in \mathbb{R}(\alpha)$ on $[a, b]$ and

$$\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$$

Or

State and prove Cauchy criterion for uniform convergence.

Unit-IV

9. State and prove Taylor's theorem.

Or

PTO

State and prove Abel's theorem.

Unit-V

10. Suppose k is a compact subset of R^n, and $\{V_\alpha\}$ is an open cover of K. Then prove that there exist *function* $\psi_1, ... \psi_5 \in \mathbb{C}(R^n)$ such that

$$\psi_1(x) + ... + \psi_5(x) = 1$$

for every $x \in K$. Moreover each ψ_i, has its support in some V_α.

Or

State and prove inverse function theorem.