ER-1402

M. A. / M. Sc. (First Semester) Examination, Nov.-Dec. 2019

MATHEMATICS

Paper : Second

(Real Analysis)

Time Allowed : Three hours

Maximum Marks : 40

Note : Attempt questions of all two sections as directed. Distribution of marks is given with sections.

Section–'A'

(Short Answer Type Questions) **5×3=15**

Note : Attempt all the questions. Each question carries 3 marks.

Unit-I

1. Define Refinement and Common Refinrment?

Or

If $U(P,f,\alpha) - L(P,f,\alpha)$ holds for some partitions P and some $\in$, then it holds with the same $\in$ for every refinement of P.

Unit-II

2. Explain integration of vector-valued function.

Or

Define rearrangement of terms of a Series with example.

Unit-III

3. If $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly on E, then f is continuous on E.

Or

State and prove M_n-test with an example.

Unit-IV

4. Define continuously differentiable functions.

Or

If f maps on open set $E \subset R^n$ into R^m 0 then φ the partial derivative $D_j f_i$ exists and are continuous on E, if $f \in C'(E)$ $(1 \le i \le m)$, $(1 \le j \le n)$.

Unit-V

5. Give the statement of inverse function theorem.

Or

Explain differential forms.

Section-'B'

(Long Answer Type Questions) **5×5=25**

Note : *Attempt all the five questions. Each question carries 5 marks.*

Unit-I

6. If P^* is a refinement of P, then

$$L(P, f, \alpha) \le L(P^*, f, \alpha)$$

$$U(P^*, f, \alpha) \le W(P, f, \alpha)$$

Or

Suppose $f \in R(\alpha)$ on $[a,b]$ $m \le f \le M, \phi$ is continuous on $[m, M]$ and $h(x)_2 \phi$ $(f(x))$ on $[a,b]$, then $h \in R(\alpha)$ on $[a,b]$.

Unit-II

7. State and prove Riemann's theorem.

Or

Show that $\{f_n\}$ coverges uniformly to a function f, where

$$f_n(x) = \frac{x}{1+nx^2} \quad (n = 1, 2, 3, ...)$$

Unit-III

8. State and prove Stone Weierstrass theorem.

Or

State and prove Cauchy criterian for uniform converngence.

Unit-IV

9. State and prove Abel's theorem.

Or

Let Ω be the set of all invertible linear operations on R^n,

If $A \in \Omega$, $B \in L(R^n)$, and

$$\| B - A \| . \| A^{-1} \| < 1$$

Then $B \in \Omega$.

Unit-V

10. State and prove Implicit function theorem.

Or

State and prove Stoke's theorem.