ER-1401

M. A./M.Sc. (First Semester) Examination, Nov.-Dec. 2019

MATHEMATICS

Paper : First

(Advanced Abstract Algebra-I)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt quesitons of all* ***two*** *sections as directed. All symbols have their usual meaning.*

Section-'A'

(Short Answer Type Questions) **5×3=15**

Note : *This section will contain* ***five*** *questions. Internal choice have been provided in each question. Each question will carry 3 marks.*

1. Define Equivalence of two subnormal series.

Or

Let H and K be two subgroups of G such that $kH = Hk$ for every $k \in K$ then prove that Hk is a subgroup of G and H is a normal subgroup of Hk.

2. Define solvable group with example.

Or

Show that symmetric group S_3 of degree 3 is solvable.

3. Define Extension field with examples.

Or

Prove that every finite extension of a field is an algebraic extension.

4. Prove that the field Q of rational numbers is a prime field.

Or

Prove that the multiplicative group of non-zero elements of a finite field is cyclic.

5. Define Galois group with example.

Or

Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over Q.

Section-'B'

(Long Answer Type Questions) 5×5=25

Note : *This section will contain five questions. Internal choice have been provided in each question. Each question will carry 5 marks.*

6. Prove that any two subnormal series of a group have equivalent refinements.

Or

Show that an abelian group G has a composition series if an only if G is finite.

7. Prove that every homomorphic image of a solvable group is solvable.

Or

Get G be a nilpotent group. Then prove that every subgroup of G and every homomorphism image of G are nilpotent.

8. Let $F \subseteq E \subseteq K$ be fields. If K is a finite extension of E and E is a finite extension of F then prove that K is a finite extension of F and $[K:F] = [K:E][E:F]$.

Or

Find the degree of splitting field of $x^4 - 5x^2 + 6$ over Q.

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9. Prove that the prime field of a field F is either isomorphic to Q or to Z/p where p is prime.

Or

Prove that any two finite fields having the same number of elements are isomorphic.

10. If $f(x) \in F[x]$ has r distinct roots in its splitting field E over F, then prove that the Galois group $G(E/F)$ of $f(x)$ is a subgroup of the symmetric group S_r

Or

Find the Galois group of $x^4 + 1 \in Q[x]$.

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